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The matrix superpropagator of a chiral invariant pion–nucleon interaction

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Abstract. We take a chiral SU(2) × SU(2) invariant pion–nucleon interaction to first order in the meson covariant derivative. The covariant derivatives are identified in terms of the Gürsey matrix, and choosing the exponential parametrization for this we calculate the matrix superpropagator using Fourier transform techniques.

1. Introduction

An often quoted example of the occurrence of nonpolynomial Lagrangians is in chiral theories, that is, when we take a nonlinear realization of the chiral symmetry on the pseudoscalar meson fields. These Lagrangians were developed as 'effective' Lagrangians to investigate, and reproduce more simply, the results of current algebra (Weinberg 1967), the chiral groups involved being SU(2) × SU(2) and SU(3) × SU(3). We denote these by K(2) and K(3) respectively.

If the Lagrangian was of the form $gF(\lambda\phi)$ or $g\bar{\psi}\psi F(\lambda\phi)$, with F nonpolynomial in the meson fields ϕ, then F was expanded to low order in the minor coupling constant λ, and the resultant polynomial interaction was used in a tree-graph approximation (Gasiorowicz and Geffen 1969 and references therein). Since then it has been suggested that we take the effective Lagrangian as a more orthodox field theory object by putting in loops (Charap 1970) and also by using the developments in nonpolynomial field theories to handle the unexpanded Lagrangian, that is, perform the perturbation series in the major coupling constant g, where for each order in g we have all orders in the minor coupling constant.

Now, in a closed form, the chiral Lagrangians can be expressed as matrix interactions. The matrices form a representation of a nonabelian group and this does not allow us to make use of the usual functional formulation of the S matrix (Delbourgo 1972). Different techniques have had to be devised and, for a K(2) or K(3) interaction expressed in terms of the Gürsey matrix with no derivatives, the two-point functions, the so called superpropagators, have been given by Delbourgo (1972) and Ashmore and Delbourgo (1971).

We here consider the K(2) pion–nucleon case when we include a derivative coupling, as we would if we followed the Weinberg (1968) prescription for writing chiral invariant Lagrangians, or if we used the Gürsey approach (Chang and Gürsey 1967) to include an arbitrary coupling constant. Our calculation of the pion superpropagator makes use of the integral transform technique of Delbourgo (1972). This method can be used for SU(2) because the pions transform under a representation of the adjoint group SU(2)/Z(2) which is isomorphic to the rotation group SO(3). This allows a change from cartesian coordinates to spherical polars, that is, to the single group invariant and two angles. We return briefly to this point in the last section.

We emphasize that in performing these calculations we are treating the fields not as operator-valued distributions but as real, c number quantities; this allows us to integrate, differentiate and form power series expansions with respect to the fields.

In § 2 we review briefly the prescriptions for writing chiral invariant Lagrangians, and the identification of the covariant derivatives in terms of the Gürsey matrix. In § 3 we give our reasons for choosing the exponential parametrization for the latter, and in §§ 4 and 5 we outline our calculation of the superpropagator. We conclude in § 6 with some remarks on the K(3) problem.

2. Chiral invariant Lagrangians

If we are to use a nonlinear realization of the chiral symmetry on the pseudoscalar mesons, which act as the massless Goldstone bosons (Gasiorowicz and Geffen 1969); then there exist two equivalent prescriptions for writing a chiral invariant Lagrangian. On one hand we have the algebraic approach, given for K(2), by Weinberg (1968) and in greater generality by Coleman *et al* (1969) and Isham (1969), and on the other hand the matrix approach associated with Gürsey (Chang and Gürsey 1967). The relationship between the two methods was first formalized by Macfarlane *et al* (1970) and recently the K(3) and general K(n) solutions have been given by Barnes *et al* (1972a).

Our work is mainly concerned with the nonpolynomial problems that are raised by these prescriptions, and as, in any case, these prescriptions are well known we will omit all proofs and just state what they are.

2.1. The Weinberg method

We start with fields which transform nonlinearly under the chiral group, and from which we construct the so called covariant derivatives. If we are interested in SU(2) pions and nucleons, and if we restrict the pion–nucleon interaction to first order in the meson derivative, we would write a chirally invariant Lagrangian with massless pions as:

$$\mathscr{L} = \mathscr{L}_\pi + \bar{\psi}^\alpha(\mathrm{i}\not{D} - m)_\alpha{}^\beta \psi_\beta + g\bar{\psi}^\alpha\gamma^\mu\gamma_5 D_\mu \boldsymbol{\pi}_\alpha{}^\beta \psi_\beta \qquad (1)$$

where $\boldsymbol{\pi} = \pi_i\boldsymbol{\sigma}_i$ and the $\boldsymbol{\sigma}_i$ are the Pauli 2×2 traceless, hermitian matrices. The pion and nucleon covariant derivatives are respectively $D_\mu\pi_i$ and $D_\mu\psi_\alpha$, and as their construction follows quite easily from the Gürsey method we leave further details until it has been outlined. The pion term is given by

$$\mathscr{L}_\pi = \tfrac{1}{4}F_\pi^2\,\mathrm{Tr}(D_\mu\boldsymbol{\pi}D^\mu\boldsymbol{\pi}) \qquad (2)$$

and contains the meson kinetic terms and then contributions to π–π scattering, where F_π is the unrenormalized pion decay constant.

In the nucleon terms we note that the nucleon covariant derivative involves pion–nucleon interactions but it will be shown in § 4 that these are restricted to an even number of pions at the vertex. It is the pion covariant derivative which gives the odd number of pions and includes the single-pion Yukawa vertex. From this we find that, if the pseudovector coupling constant is $G/2m$, then

$$g = \frac{GF_\pi}{2m}.$$

2.2. *The Gürsey method*

We start with nucleon fields N_α which transform linearly under the chiral group, that is, under an SU(2) transformation

$$N_\alpha \to [\exp(i\theta_j \boldsymbol{\sigma}_j)]_\alpha{}^\beta N_\beta \tag{3a}$$

and under the parity changing part of K(2) we demand

$$N_\alpha \to [\exp(i\phi_j \boldsymbol{\sigma}_j \gamma_5)]_\alpha{}^\beta N_\beta \tag{3b}$$

where θ_i and ϕ_i are the parameters of the transformation and we have suppressed the Dirac labels.

Then a nucleon kinetic term $i\bar{N}\partial\!\!\!/ N$ is K(2) invariant but a mass term $m\bar{N}N$, though SU(2) invariant, is not K(2) invariant. To overcome this, Gürsey introduced a two by two unitary matrix† $\hat{\boldsymbol{U}}$ which is a nonlinear function of $i\beta\gamma_5\boldsymbol{\pi}$. (The dimensional constant β is such that $\beta\pi$ is dimensionless.) The Gürsey matrix $\hat{\boldsymbol{U}}$ is constructed such that $m\bar{N}\hat{\boldsymbol{U}}N$, now nonlinear in the pions, is K(2) invariant and, amongst other things, this imposes unimodularity on $\hat{\boldsymbol{U}}$.

With the observation, attributed by Gürsey to Ogievetskii, that there exists the chirally invariant pion–nucleon coupling $\bar{N}(\hat{\boldsymbol{U}}\partial\!\!\!/\hat{\boldsymbol{U}})N$ which is to first order in the meson derivatives we can write the Gürsey equivalent of equation (1):

$$\mathscr{L} = \mathscr{L}_\pi + \bar{N}^\alpha(i\partial\!\!\!/ - m\hat{\boldsymbol{U}})_\alpha{}^\beta N_\beta + ig'\bar{N}^\alpha(\hat{\boldsymbol{U}}\partial\!\!\!/\hat{\boldsymbol{U}})_\alpha{}^\beta N_\beta \tag{4}$$

where

$$\mathscr{L} = \tfrac{1}{4}F_\pi{}^2\,\mathrm{Tr}(\partial_\mu\hat{\boldsymbol{U}}\partial^\mu\hat{\boldsymbol{U}}^\dagger). \tag{5}$$

At this stage we can identify the covariant derivatives in equation (2). We first make the redefinition of nucleon fields

$$\psi_\alpha \equiv [\hat{\boldsymbol{U}}^{1/2}]_\alpha{}^\beta N_\beta \tag{6}$$

where we take the unitary, unimodular square root. These fields now transform nonlinearly (in the pions) under the chiral group and are the type of field that the Weinberg approach starts with.

We also remove the γ_5 dependence of $\hat{\boldsymbol{U}}$ by using

$$\hat{\boldsymbol{U}} = P_+\boldsymbol{U} + P_-\boldsymbol{U}^\dagger$$

where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ are projection operators in the Dirac space. The matrix $\boldsymbol{U}$ is the same function of $i\beta\boldsymbol{\pi}$ as $\hat{\boldsymbol{U}}$ was of $i\beta\gamma_5\boldsymbol{\pi}$.

We then find we have:

$$i\bar{N}(\hat{\boldsymbol{U}}\partial\!\!\!/\hat{\boldsymbol{U}})N = \bar{\psi}\gamma^\mu\gamma_5 D_\mu\boldsymbol{\pi}\psi \tag{7a}$$

$$i\bar{N}\partial\!\!\!/ N = i\bar{\psi}D\!\!\!\!/\,\psi - \tfrac{1}{2}\bar{\psi}\gamma^\mu\gamma_5 D_\mu\boldsymbol{\pi}\psi \tag{7b}$$

where the covariant derivatives have been identified as

$$D_\mu\boldsymbol{\pi}_\alpha{}^\beta = i\{\boldsymbol{V}^\dagger, \partial_\mu\boldsymbol{V}\}_\alpha{}^\beta \tag{8a}$$

$$D_\mu\psi_\alpha = \partial_\mu\psi_\alpha + \tfrac{1}{2}[\boldsymbol{V}^\dagger, \partial_\mu\boldsymbol{V}]_\alpha{}^\beta\psi_\beta \tag{8b}$$

and $\boldsymbol{V} \equiv \boldsymbol{U}^{1/2}$.

† Strictly, $\hat{\boldsymbol{U}}$ is an 8×8 matrix in the direct product space of the 2×2 I-spin matrices and the 4×4 Dirac matrices. We will consistently leave the Dirac matrices to take care of themselves, and any traces indicated are to be taken over the SU(2) matrix space.

Using equations (7) and (8) we find that, as we expect, the invariant Lagrangians given in equations (1) and (4) are equivalent, with $g = (g' - \frac{1}{2})$.

We can rewrite equation (1) as

$$\mathscr{L} = \mathscr{L}_\pi + \bar{\psi}^\alpha(\mathrm{i}\partial\!\!\!/ - m)\psi_\alpha + \mathscr{L}^{\pi\mathrm{N}}_{\mathrm{int}} \tag{9}$$

where the pion–nucleon interaction is given by

$$\mathscr{L}^{\pi\mathrm{N}}_{\mathrm{int}} = \mathrm{i}\bar{\psi}^\alpha\gamma^\mu(\tfrac{1}{2}[\boldsymbol{V}^\dagger, \partial_\mu \boldsymbol{V}] + g\gamma_5\{\boldsymbol{V}^\dagger, \partial_\mu \boldsymbol{V}\})_\alpha{}^\beta\psi_\beta \equiv \mathrm{i}\bar{\psi}^\alpha\gamma^\mu \boldsymbol{X}_\mu(\pi)_\alpha{}^\beta\psi_\beta. \tag{10}$$

This is the matrix interaction which we would like to investigate in perturbation theory.

3. The exponential parametrization

It is well known that Weinberg's solution for the covariant derivatives involves an arbitrariness expressible as a redefinition of the pion fields, and we would expect this also to be true of the matrix solution if the two are equivalent. This is indeed the case, because we have the arbitrariness in the matrix function $\boldsymbol{U}(\pi)$. We will first decide on a method to parametrize $\boldsymbol{U}$ and then pick a particular function of $\boldsymbol{\pi}$ with which to perform the calculations of the next section.

Two common methods of parametrizing a unitary matrix involve a hermitian matrix $\boldsymbol{H}$. We have the exponential parametrization $\mathrm{e}^{\mathrm{i}\boldsymbol{H}}$ and the Cayley, or rational method $(1+\mathrm{i}\boldsymbol{H})(1-\mathrm{i}\boldsymbol{H})^{-1}$. If we further require unimodularity then the exponential case requires $\boldsymbol{H}$ traceless and for 2×2 matrices this is also true for the rational case. Thus we can write $\boldsymbol{H} = h_i\boldsymbol{\sigma}_i$ for some real $\{h_i\}$.

Our problem is to parametrize the unitary, unimodular Gürsey matrix $\boldsymbol{U}$, and more importantly for our interests its square root $\boldsymbol{V}$, in terms of the pion fields. Any SU(2) vector formed from the π_i can only be proportional to π_i, so that we must have $h_i = \alpha(\pi)\pi_i$, where $\alpha(\pi)$ is some arbitrary scalar function of π_i and is therefore a function of the only SU(2) invariant $\pi \equiv (\pi_i\pi_i)^{1/2}$. We have

$$\begin{aligned} \boldsymbol{U} &= \exp \mathrm{i}\alpha(\pi)\boldsymbol{\pi} \\ \boldsymbol{V} &= \exp \tfrac{1}{2}\mathrm{i}\alpha(\pi)\boldsymbol{\pi} \end{aligned} \tag{11}$$

or

$$\begin{aligned} \boldsymbol{U} &= (1+\mathrm{i}\alpha(\pi)\boldsymbol{\pi})(1-\mathrm{i}\alpha(\pi)\boldsymbol{\pi})^{-1} = \frac{1-\alpha^2\pi^2+2\mathrm{i}\alpha\boldsymbol{\pi}}{1+\alpha^2\pi^2} \\ \boldsymbol{V} &= \frac{1+\mathrm{i}\alpha(\pi)\boldsymbol{\pi}}{\sqrt{(1+\alpha^2\pi^2)}} \end{aligned} \tag{12}$$

where we have made use of the product law $\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j = \delta_{ij} + \mathrm{i}\epsilon_{ijk}\boldsymbol{\sigma}_k$.

At this point we note that, provided α is an even function, then the parametrization (11) can give entire functions in π_i, whereas (12) cannot; we have singularities at $\pi\alpha(\pi) = \mathrm{i}$ (for α odd both involve singularities).

The importance of this is that nonpolynomial interactions which are entire functions of the fields seem to have many advantages† over the non-entire interactions; not least being that the former define strictly local field theories in the sense of Jaffe (1967).

† We recommend the review of advantages given by Salam (1971) and Isham *et al* (1971).

Furthermore, if we took the particular choice of $\alpha(\pi) = 2\lambda$, a dimensional constant, then we have an interaction involving $\exp(\mathrm{i}\lambda\boldsymbol{\pi})$ type terms which we might expect to give terms like $\exp(-\lambda^2\Delta)$, with some matrix complications, in the superpropagator. This, in fact, will be the case and it is precisely this function of the free field propagator, Δ, which Lehmann and Pohlmeyer (1971) have interpreted as a uniquely defined distribution.

We will play safe, therefore, and take

$$V = \exp(\mathrm{i}\lambda\boldsymbol{\pi}). \tag{13}$$

There is yet another benefit accruing from this choice, which is involved with the practical matter of simplifying matrix complications. If we write a general $V = a + \mathrm{i}b_j\boldsymbol{\sigma}_j$ then we can identify $a = \frac{1}{2}\operatorname{Tr} V$, and $\mathrm{i}b_j = \frac{1}{2}\operatorname{Tr}(\boldsymbol{\sigma}_j V)$. For the choice of equation (13) we have

$$\mathrm{i}b_j = -\frac{\mathrm{i}}{\lambda}\frac{\partial}{\partial\pi_j}\frac{1}{2}\operatorname{Tr} V$$

so that

$$V = \frac{1}{2}\left(1 - \frac{\mathrm{i}}{\lambda}\boldsymbol{\sigma}_j\frac{\partial}{\partial\pi_j}\right)\operatorname{Tr} V \tag{14}$$

and this will enable us to separate the matrix part of the calculation from the combinatorics of the T products. $\operatorname{Tr} V$ is a scalar function of π and for the choice (13) $\operatorname{Tr} V = 2\cos\lambda\pi$.

4. The superpropagator

If we are interested in a perturbation theory for the nonpolynomial interaction given by equation (10), then the first object that we will encounter is the two-point function for the exchange of any number of mesons, the superpropagator, given by

$$\underset{\mu\nu}{\Sigma(\Delta)}{}_{\alpha}{}^{\beta}{}_{\alpha}{}^{\delta} = \langle 0|T^* : \boldsymbol{X}_\mu(\pi)_\alpha{}^\beta : : \boldsymbol{X}_\nu(\pi')_\gamma{}^\delta :|0\rangle \tag{15}$$

where $\pi_i' = \pi_i(x')$. The free field propagator Δ is given by

$$\langle 0|T[\pi_i(x)\pi_j(x')]|0\rangle \equiv \langle \pi_i(x), \pi_j(x')\rangle \equiv \delta_{ij}\Delta(x-x').$$

The calculation of the superpropagator involves two tasks; one is the handling of the matrix complication which means that, although $\langle \exp(\xi_i\pi_i), \exp(\xi_j'\pi_j')\rangle = \exp(\xi_i\xi_i'\Delta)$ for scalar parameters ξ_i, we have $\langle \exp\boldsymbol{\pi}, \exp\boldsymbol{\pi}'\rangle \neq \exp(\boldsymbol{\sigma}_i\boldsymbol{\sigma}_i\Delta)$; and the other will be the combinatoric problems raised by taking the T products of nonpolynomial functions of multiplets of fields which are not the simple exponentials $\exp(\xi_i\pi_i)$.

The expression (14) will enable us to separate the matrix part, and the combinatoric problems will be obviated by replacing nonpolynomial functions by Fourier transform representations where all the field dependence is in an exponential; the application of Wick's theorem will then give back exponential functions. We have

$$\boldsymbol{X}_\mu(\pi)_\alpha{}^\beta = \tfrac{1}{2}[V^\dagger, \partial_\mu V]_\alpha{}^\beta + g\gamma_5\{V^\dagger, \partial_\mu V\}_\alpha{}^\beta \equiv \boldsymbol{A}_\mu(\pi) + g\gamma_5\boldsymbol{B}_\mu(\pi). \tag{16}$$

Then, using expression (14), we find

$$A_\mu = \frac{\mathrm{i}}{4\lambda^2}\boldsymbol{\sigma}_i\epsilon_{ijk}\partial_j \operatorname{Tr} V\partial_\mu\partial_k \operatorname{Tr} V \tag{17a}$$

$$B_\mu = \frac{\mathrm{i}}{2\lambda}\boldsymbol{\sigma}_j(\partial_j \operatorname{Tr} V\partial_\mu \operatorname{Tr} V - \operatorname{Tr} V\partial_\mu\partial_j \operatorname{Tr} V) \tag{17b}$$

where $\partial_j = \partial/\partial\pi_j$ and $\operatorname{Tr} V = 2\cos\lambda\pi$.

If we now use

$$\partial_j f(\pi) = \frac{\pi_j}{\pi}\frac{\partial}{\partial\pi}f(\pi) \qquad \text{and} \qquad \partial_\mu f(\pi) = \partial_\mu\pi_j\partial_j f(\pi)$$

we have

$$A_\mu = \mathrm{i}\boldsymbol{\sigma}_i\epsilon_{ijk}\pi_j\partial_\mu\pi_k T^{(1)}(\pi) \tag{18a}$$

and

$$B_\mu = \mathrm{i}\boldsymbol{\sigma}_j(T^{(2)}(\pi)\partial_\mu\pi_j + T^{(3)}(\pi)\pi_k\partial_\mu\pi_k\pi_j) \tag{18b}$$

where

$$T^{(1)}(\pi) = \pi^{-2}\sin^2\lambda\pi$$
$$T^{(2)}(\pi) = \pi^{-1}\sin 2\lambda\pi$$
$$T^{(3)}(\pi) = \pi^{-3}(2\lambda\pi - \sin 2\lambda\pi) \tag{19}$$

are all entire functions of π.

We can see here one major simplification in calculating the superpropagator. Because A_μ is an even function, and B_μ is an odd function of the pion fields, we will have the term $\langle :A_\mu(\pi):, :B_\nu(\pi'):\rangle$ vanishing. We are left with the $\langle A, A\rangle$ and $\langle B, B\rangle$ terms, equation (15) being of the form

$$\underset{\mu\nu}{\Sigma}(\Delta) = 1\otimes 1\langle A_\mu(\pi), A_\nu(\pi')\rangle + g^2\gamma_5\otimes\gamma_5\langle B_\mu(\pi), B_\nu(\pi')\rangle \tag{20}$$

where the direct products of the Dirac matrices are shown.

In the next section we will illustrate in some detail the method of calculating the $\langle A, A\rangle$ terms, but just state the result for the $\langle B, B\rangle$ term as no different technique is used.

5. The calculation

Contributing to equation (20) we have the term

$$\langle :A_\mu(\pi):, :A_\nu(\pi'):\rangle = -\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_c\epsilon_{ijk}\epsilon_{abc}F^{jkab}_{\mu\nu}(\Delta)$$

where

$$F^{jkab}_{\mu\nu}(\Delta) = \langle :T^{(1)}(\pi)\pi_j\partial_\mu\pi_k:, :T^{(1)}(\pi')\pi'_a\partial_\nu\pi'_b:\rangle. \tag{21}$$

The SU(2) properties are taken care of by noting that, as the fields in the T product are eliminated in pairs, then, in general, objects like $F^{jk\ldots l}_{\mu\nu}$ can only be SU(2) invariant numerical tensors built from Kronecker deltas. In this case we have a fourth rank tensor which must be expressible as the linear combination

$$F^{jkab}_{\mu\nu} = \sum_{r=1}^{3} F^{(r)}_{\mu\nu}\alpha(r)^{jkab} \tag{22}$$

for scalar functions $F^{(r)}_{\mu\nu}$ and the $\alpha(r)$ are the three independent tensors given by

$$\alpha(1)_{jkab} = \delta_{jk}\delta_{ab}$$
$$\alpha(2)_{jkab} = \delta_{ja}\delta_{kb}$$
$$\alpha(3)_{jkab} = \delta_{jb}\delta_{ka}.$$

Using (22) then gives that

$$\epsilon_{ijk}\epsilon_{abc}F^{jkab}_{\mu\nu} = \tfrac{1}{3}\delta_{ic}(\alpha(2)-\alpha(3))_{jkab}F^{jkab}_{\mu\nu}. \tag{23}$$

We now introduce the Fourier transform representation

$$T^{(1)}(\pi) = \int_{-\infty}^{+\infty} d^3\xi \tilde{T}^{(1)}(\xi)\exp(i\pi_j\xi_j) \tag{24}$$

where the triple integral is over the three parameters $\{\xi_i\}$. All the pion field dependence is in the exponential, so applying (24) to (21) will leave us with integrals over the much simpler T product given by

$$G^{jkab}_{\mu\nu}(\Delta) = \langle :\exp(i\pi_l\xi_l)\pi_j\partial_\mu\pi_k:, :\exp(i\pi'_m\xi'_m)\pi'_a\partial_\nu\pi'_b:\rangle. \tag{25}$$

Expanding the exponentials as power series, using Wick's theorem and resumming gives

$$(\alpha(2)-\alpha(3))_{jkab}G^{jkab}_{\mu\nu} = 2(\partial_\mu\Delta\partial_\nu\Delta-\Delta\partial_\mu\partial_\nu\Delta)(3-\Delta\xi_l\xi'_l)\exp(-\Delta\xi_l\xi'_l). \tag{26}$$

Replacing $\xi_l\xi'_l$ by $-\partial/\partial\Delta$ acting on the exponential we are left with just the following integral to perform:

$$I \equiv \int_{-\infty}^{\infty} d^3\xi \int_{-\infty}^{\infty} d^3\xi' \tilde{T}^{(1)}(\xi)\tilde{T}^{(1)}(\xi')\exp(-\Delta\xi_l\xi'_l). \tag{27}$$

Using the inverse relation to (24) we can rewrite the above in terms of the original functions $T^{(1)}(u_i)$. Then, changing to polar coordinates and using the fact that the $T^{(1)}$ are scalar functions of $u = \sqrt{(u_iu_i)}$ we can perform the angular integrations to arrive at

$$I = \frac{2}{\pi}\int_0^{\infty} du \int_0^{\infty} dv\ T^{(1)}(v)T^{(1)}(i\Delta u)uv\sin(uv). \tag{28}$$

These integrals are easily done, and if we define the dimensionless quantity $Z \equiv \lambda^2\Delta$, we have

$$I = \frac{\lambda^4}{2}Z^{-2}C(Z) \tag{29}$$

where

$$C(Z) = \int_0^{2Z}\frac{dx}{x}\sinh^2 x = \tfrac{1}{2}\,\mathrm{chi}(4Z) - \tfrac{1}{2}\ln(4\gamma Z).$$

Thus, combining equations (21), (23), (26) and (29) we have

$$\langle :A_\mu(\pi):, :A_\nu(\pi'):\rangle = \tfrac{1}{3}\boldsymbol{\sigma}_i\otimes\boldsymbol{\sigma}_i(C(Z)+\sinh^2(2Z))\partial_\mu\partial_\nu\ln Z \tag{30}$$

where we have used the formal identity

$$\partial_\mu \partial_\nu \ln Z = Z^{-2}(Z\partial_\mu\partial_\nu Z - \partial_\mu Z \partial_\nu Z).$$

The calculation of the $\langle \boldsymbol{B}_\mu, \boldsymbol{B}_\nu \rangle$ term involves more work but follows a similar pattern to yield

$$\langle :\boldsymbol{B}_\mu(\pi):, :\boldsymbol{B}_\nu(\pi'):\rangle = \tfrac{4}{3}\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_i (4ZC(Z)\partial_\mu\partial_\nu \ln Z + 3\partial_\mu\partial_\nu Z). \tag{31}$$

Thus our final result for the pion superpropagator of the pion–nucleon interaction is given by

$$\underset{\mu\nu}{\Sigma(Z)}{}_\alpha{}^\beta{}_\gamma{}^\delta = \tfrac{1}{3}(\boldsymbol{\sigma}_i)_\alpha{}^\beta(\boldsymbol{\sigma}_i)_\gamma{}^\delta \underset{\mu\nu}{H(Z)}$$

where

$$\underset{\mu\nu}{H(Z)} = 1\otimes 1(C(Z)+\sinh^2(2Z))\partial_\mu\partial_\nu \ln Z + 4g^2\gamma_5\otimes\gamma_5(3\partial_\mu\partial_\nu Z + 4ZC(Z)\partial_\mu\partial_\nu \ln Z) \tag{32}$$

and $Z(x) = \lambda^2\Delta(x)$.

We have checked to order λ^6 that the above expression agrees with that obtained directly from (15) by expanding $\boldsymbol{X}_\mu(\pi)$ to low order in λ.

6. The K(3) problem

We conclude this paper with some remarks on the equivalent K(3) problem. The direct extension of the results in § 2 to K(3) would involve the octet of pseudoscalar mesons M_i and the three quarks, but only needs slight modification to incorporate the octet of spin-half baryons. The general form of the SU(3) Gürsey matrix is available (Barnes *et al* 1972b) but, in any case, we could again pick the exponential solution and proceed with the calculation as before.

The main problem comes with trying to perform the SU(3) equivalent of integrals like that occurring in (27), because the traces would now be functions of the *two* SU(3) Casimir invariants. This means that we cannot use the coordinate change from the cartesian M_i to eight-dimensional spherical polars with their single invariant; that is, the adjoint group SU(3)/Z(3) under which the M_i transform is isomorphic to a *subgroup* of the eight-dimensional rotation group SO(8). The SU(3) equivalent of the spherical polars (two invariants and six angles) does exist (Charap and Davies 1972) but so far has proved too complex to allow the integrals to be performed.

It may be that the matrix methods of Ashmore and Delbourgo (1971) can be extended to calculate the equivalent K(3) superpropagator.

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References

Abdus Salam 1971 *AIP Conf. Proc.* **2** 259–74
Ashmore J and Delbourgo R 1971 *J. math. Phys.* to be published

Barnes K J, Dondi P H and Sarkar S C 1972a *Proc. R. Soc.* A to be published
—— 1972b *J. Phys.* A: *Gen. Phys.* **5** 555–62
Chang P and Gürsey F 1967 *Phys. Rev.* **164** 1752–61
Charap J M 1970 *Phys. Rev.* D **2** 1554–61
Charap J M and Davies P T 1972 *J. Phys.* A: *Gen. Phys.* **5** 563–8
Coleman S, Wess J and Zumino B 1969 *Phys. Rev.* **177** 2239–47
Delbourgo R 1972 *J. math. Phys.* **13** 464–8
Gasiorowicz S and Geffen D A 1969 *Rev. mod. Phys.* **41** 531–73
Isham C J 1969 *Nuovo Cim.* A **59** 356–76
Isham C J, Abdus Salam and Strathdee J 1971 *Trieste Preprint* IC/71/14
Jaffe A 1967 *Phys. Rev.* **158** 1454–61
Lehmann H and Pohlmeyer K 1971 *Commun. math. Phys.* **20** 101–10
Macfarlane A J, Sudbery A and Weisz P 1970 *Proc. R. Soc.* A **314** 217–50
Weinberg S 1967 *Phys. Rev. Lett.* **18** 188–91
—— 1968 *Phys. Rev.* **166** 1568–77